OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4728

Mechanics 1

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

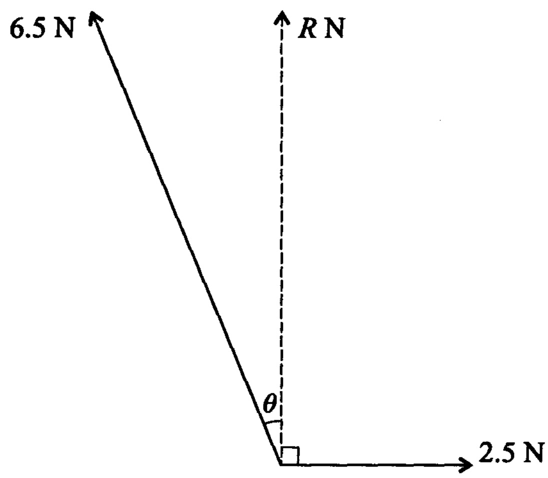
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 Each of two wagons has an unloaded mass of 1200 kg. One of the wagons carries a load of mass m kg and the other wagon is unloaded. The wagons are moving towards each other on the same rails, each with speed $3\ \mathrm{m\,s^{-1}}$, when they collide. Immediately after the collision the loaded wagon is at rest and the speed of the unloaded wagon is $5\ \mathrm{m\,s^{-1}}$. Find the value of m. [5]

2



Forces of magnitudes 6.5 N and 2.5 N act at a point in the directions shown. The resultant of the two forces has magnitude R N and acts at right angles to the force of magnitude 2.5 N (see diagram).

(i) Show that $\theta = 22.6°$, correct to 3 significant figures. [3]

(ii) Find the value of R. [3]

3 A man travels 360 m along a straight road. He walks for the first 120 m at $1.5\ \mathrm{m\,s^{-1}}$, runs the next 180 m at $4.5\ \mathrm{m\,s^{-1}}$, and then walks the final 60 m at $1.5\ \mathrm{m\,s^{-1}}$. The man's displacement from his starting point after t seconds is x metres.

(i) Sketch the (t, x) graph for the journey, showing the values of t for which $x = 120$, 300 and 360. [5]

A woman jogs the same 360 m route at constant speed, starting at the same instant as the man and finishing at the same instant as the man.

(ii) Draw a dotted line on your (t, x) graph to represent the woman's journey. [1]

(iii) Calculate the value of t at which the man overtakes the woman. [5]

4 A cyclist travels along a straight road. Her velocity $v\,\mathrm{m\,s^{-1}}$, at time t seconds after starting from a point O, is given by

$$v = 2 \quad \text{for } 0 \leqslant t \leqslant 10,$$
$$v = 0.03t^2 - 0.3t + 2 \quad \text{for } t \geqslant 10.$$

(i) Find the displacement of the cyclist from O when $t = 10$. [1]

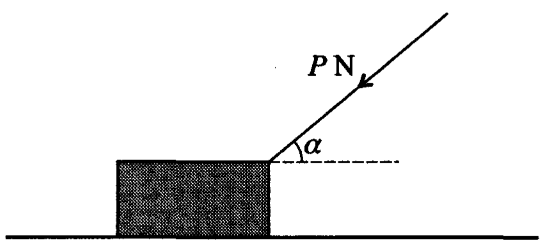
(ii) Show that, for $t \geqslant 10$, the displacement of the cyclist from O is given by the expression $0.01t^3 - 0.15t^2 + 2t + 5$. [4]

(iii) Find the time when the acceleration of the cyclist is $0.6\,\mathrm{m\,s^{-2}}$. Hence find the displacement of the cyclist from O when her acceleration is $0.6\,\mathrm{m\,s^{-2}}$. [5]

5 A block of mass m kg is at rest on a horizontal plane. The coefficient of friction between the block and the plane is 0.2.

(i) When a horizontal force of magnitude 5 N acts on the block, the block is on the point of slipping. Find the value of m. [3]

(ii)



When a force of magnitude P N acts downwards on the block at an angle α to the horizontal, as shown in the diagram, the frictional force on the block has magnitude 6 N and the block is again on the point of slipping. Find

(a) the value of α in degrees,

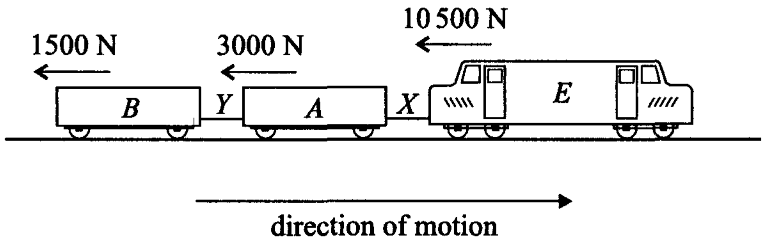
(b) the value of P.

[8]

[Questions 6 and 7 are printed overleaf.]

[Turn over

6



A train of total mass 80 000 kg consists of an engine E and two trucks A and B. The engine E and truck A are connected by a rigid coupling X, and trucks A and B are connected by another rigid coupling Y. The couplings are light and horizontal. The train is moving along a straight horizontal track. The resistances to motion acting on E, A and B are 10 500 N, 3000 N and 1500 N respectively (see diagram).

(i) By modelling the whole train as a single particle, show that it is decelerating when the driving force of the engine is less than 15 000 N. [2]

(ii) Show that, when the magnitude of the driving force is 35 000 N, the acceleration of the train is $0.25\,\text{m s}^{-2}$. [2]

(iii) Hence find the mass of E, given that the tension in the coupling X is 8500 N when the magnitude of the driving force is 35 000 N. [3]

The driving force is replaced by a braking force of magnitude 15 000 N acting on the engine. The force exerted by the coupling Y is zero.

(iv) Find the mass of B. [5]

(v) Show that the coupling X exerts a forward force of magnitude 1500 N on the engine. [2]

7 A particle of mass 0.1 kg is at rest at a point A on a rough plane inclined at $15°$ to the horizontal. The particle is given an initial velocity of $6\,\text{m s}^{-1}$ and starts to move up a line of greatest slope of the plane. The particle comes to instantaneous rest after 1.5 s.

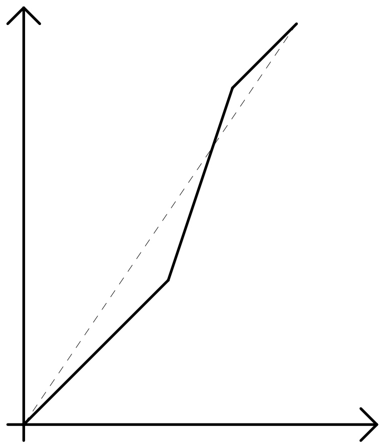
(i) Find the coefficient of friction between the particle and the plane. [7]

(ii) Show that, after coming to instantaneous rest, the particle moves down the plane. [2]

(iii) Find the speed with which the particle passes through A during its downward motion. [6]

1		Momentum before = $3M - 1200\times3$	B1	Ignore g if included; accept inconsistent directions
		Momentum after = 1200×5	B1	
				(or loss of momentum of loaded wagon = $3M$ B1 gain of momentum of unloaded wagon = $1200(5 + 3)$ B1)
		$3M – 3600 = 6000$	M1	Equation with all terms; accept with g
		$3(1200 + m) –3600 = 6000$	A1	For any correct equation in m, M
		$m = 2000$	A1 5	

2	**(i)**		M1	For resolving forces in the **i** direction or for relevant use of trigonometry
		$2.5 = 6.5\sin\theta$	A1	
		$\theta = 22.6^o$	A1 3	AG Accept verification
	(ii)		M1	For resolving forces in the **j** direction or for using Pythagoras or relevant trigonometry.
		$R = 6.5\cos22.6^o$	A1	
		$R = 6$	A1 3	

3	**(i)**		B1 B1 B1		Line segment *AB* (say) of +ve slope from origin Line segment *BC* (say) of steeper +ve slope and shorter time interval than those for *AB*. ***SR***: If the straight line segments are joined by curves, this B1 mark is not awarded Line segment *CD* (say) of less steep slope compared with *BC*. (An (x, t) graph is accepted and the references to more/less steep are reversed.)
		Time intervals 80, 40, 40 t = 80, 120, 160	B1 B1		May be implied; any 2 correct
	(ii)	Line joining (0, 0) and (160, 360)	B1 ft	6	
	(iii)	v = 360/160	M1 M1		Woman's velocity (= 2.25) For equation of man's displacement in relevant interval
		$s = 120 + 4.5(t - 80)$	A1		Accept omission of -80
		$2.25t$	M1		Woman's displacement, awarded even if t is interpreted differently in man's expression
		$t = 106\frac{2}{3}$ (107)	A1	5	Accept also 106.6, 106.7 but not 106
		SR Construction method Plotting points on graph paper t between 104 and 109 inclusive	 M1 A1		Candidates reading the <u>displacement</u> intersection from graph, then dividing this distance by the woman's speed to find t, also get v = 360/160 M1 as above for the woman's velocity.

4	**(i)**	Displacement is 20 m	B1	1	20+c (from integration) B0
	(ii)		M1		For using $s(t) = \int v(t)dt$
		$s(t) = 0.01t^3 - 0.15t^2 + 2t$ (+A)	A1		Can be awarded prior to cancelling
		$10 - 15 + 20 + A = 20$	M1		For using s(10) = cv (20)
		Displacement is $0.01t^3 - 0.15t^2 + 2t + 5$	A1	4	AG
	(iii)		M1		For using $a(t) = \mathrm{d}v/\mathrm{d}t$
		$a = 0.06t - 0.3$	A1		
		$0.06t - 0.3 = 0.6$	DM1		For starting solving $a(t)$ = 0.6 depends on previous M1
		$t = 15$	A1		
		Displacement is 35 m	B1	5	

5	**(i)**		M1		For using $F = 5$ and $F = \mu R$
		$R = mg$	M1		
		$m = 2.55$	A1	3	Accept 2.5 or 2.6
	(ii)a	$P\cos\alpha = 6$	B1		
			M1		For resolving vertically with 3 distinct forces
		$R = P\sin\alpha + 25$	A1ft		Or $P\sin\alpha$ + (cv m)g
		$0.2R = 6$	B1		For using $F = 6$ and $F = \mu R$. Can be implied by $0.2(P\sin\alpha + 25) = 6$
		$0.2(P\sin\alpha + 25) = 6$	M1		For an equation in $P\sin\alpha$ (=5)after elimination of R
		$\alpha = 39.8^o$	A1		Accept a r t 40^o
	(ii)b	$P^2 = 6^2 + 5^2$ or $P\cos 39.8^o = 6$ or $P\sin 39.8^o = 5$	M1		For eliminating or substituting for α with cv(6). Evidence is needed that 5 is the value of $P\sin\alpha$ (rather than the original frictional force)
		$P = 7.81$	A1	8	Accept a r t 7.8

6	**(i)**	10500 + 3000 + 1500	M1		For summing 3 resistances
		Driving force below 15000 gives retardation	A1	2	Accept generalised case or specific instance
	(ii)	35000 – 15000 = 80000a	M1		Newton's second law for whole train
		Acceleration is 0.25 ms^{-2}	A1	2	AG Accept verification
	(iii)		M1		For applying Newton's second law to E only, at least 2 forces out of the relevant 3.
		35000 – 10500 – 8500 = 0.25m	A1		
		Mass is 64000 kg	A1	3	
	(iv)		M1		For applying Newton's second law with all appropriate forces
		-15000 – 15000 = 80000a OR -3000-10500-15000=(80000 - m)a	A1		$a = -0.375$
			M1		For applying Newton's second law to B only, only 1 force
		-1500 = ma	A1		Or cv(a)
		Mass is 4000 kg	A1	5	
	(v)	-15000 – 10500 $\pm$ T = 64000(-0.375)	B1ft		Follow through cv (m_E, a), or accept use of m_E, a
		$T = \pm$ 1500 ➔ forward force on E of 1500 N	B1	2	
		OR (working with A and B) -1500 – 3000 $\pm$ T = (80000 - 64000)(-0.375)	B1ft		Follow through cv (m_E, a), or accept use of m_E, a
		$T = \pm$ 1500 ➔ forward force on E of 1500	B1		

7	**(i)**	$0 = 6 + (\pm)1.5a$ $a = (\mp)4\text{ms}^{-2}$ $-mg\sin15^o - F = ma$ $-0.1\times9.8\sin15^o - F = 0.1\times(-4)$ $R = 0.1g\cos15^o$ $0.146357 \ldots = \mu\, 0.946607 \ldots$ Coefficient is 0.155	M1 A1 M1 A1 B1 M1 A1	7	For using $v = u + at$ with $v = 0$ For applying Newton's second law with 2 forces For using $F = \mu R$ Anything between 0.15 and 0.16 inclusive
	(ii)	$mg\sin15^o > \mu\, mg\cos15^o$ (or $\tan 15^o > \mu$) ➔ particle moves down	M1 A1	2	For comparing weight component with frictional force (or tan 'angle of friction' with μ) Awarded if conclusion is correct even though values are wrong
	(iii)	$(6 + 0)\div2 = s\div1.5$ $s = 4.5$ $mg\sin15^o - F = ma$ $0.25364 \ldots - 0.146357 \ldots = 0.1a$ $v^2 = 2(1.07285 \ldots)4.5$ Speed is 3.11 ms^{-1}	M1 A1 M1 A1 M1 A1	6	For using $(u + v)\div2 = s\div t$ For using Newton's second law with 2 forces Values must be correct even if not explicitly stated. Note that the correct value of friction may legitimately arise from a wrong value of μ and a wrong value of R For using $v^2 = 2as$ with any value of a Accept anything rounding to 3.1 from correct working